

August 6, 1996

LBNL- 39204  
hep-ph/9608324**Gauge invariant formulation of strong  $WW$  scattering**<sup>1</sup>Michael S. Chanowitz<sup>2</sup>*Theoretical Physics Group  
Ernest Orlando Lawrence Berkeley National Laboratory  
University of California  
Berkeley, California 94720***Abstract**

Models of strong  $WW$  scattering in the  $s$ -wave can be represented in a gauge invariant fashion by defining an effective scalar propagator that represents the strong scattering dynamics. The  $\sigma(qq \rightarrow qqWW)$  signal may then be computed in U-gauge from the complete set of tree amplitudes, just as in the standard model, without using the effective  $W$  approximation (EWA). The U-gauge “transcription” has a wider domain of validity than the EWA, and it provides complete distributions for the final state quanta, including experimentally important jet distributions that cannot be obtained from the EWA. Starting from the usual formulation in terms of unphysical Goldstone boson scattering amplitudes, the U-gauge transcription is verified by using BRS invariance to construct the complete set of gauge and Goldstone boson amplitudes in  $R_\xi$  gauge.

---

<sup>1</sup>This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

<sup>2</sup>Email: chanowitz@lbl.gov

## Disclaimer

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial products process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof, or The Regents of the University of California.

*Lawrence Berkeley National Laboratory is an equal opportunity employer.*

## Introduction

The traditional approach to strong  $WW$  scattering begins by assuming a model for the scattering of the corresponding unphysical Goldstone bosons, and then uses the equivalence theorem (ET) and the effective  $W$  approximation (EWA) to compute the cross section for strong production of the longitudinally polarized gauge bosons,  $\sigma(qq \rightarrow qqW_LW_L)$ . [1] In this paper I present a gauge invariant formulation for  $s$ -wave strong  $W_LW_L$  scattering amplitudes ( $L$  denotes longitudinal polarization) that allows the  $\sigma(qq \rightarrow qqW_LW_L)$  signal to be computed directly from the complete set of tree amplitudes without using the EWA. The strong dynamics is carried by an effective scalar propagator, and the complete  $qq \rightarrow qqWW$  tree amplitude can be computed in any covariant  $R_\xi$  gauge or in unitary gauge. This formulation is more accurate and provides more information than the traditional method using the EWA.

Both approaches begin with a model Goldstone boson scattering amplitude,  $\mathcal{M}_R^X(w w \rightarrow w w)$ , required to obey unitarity and the low energy theorems of chiral symmetry. ( $X$  labels the model,  $R$  denotes a covariant renormalizable gauge, most appropriately Landau gauge, and  $w_i$  is the unphysical Goldstone boson corresponding to gauge boson  $W_{iL}$ .) The equivalence theorem [2] asserts the equality of  $\mathcal{M}_R^X(w w)$  at high energy to the corresponding amplitude of longitudinally polarized gauge bosons  $W_L$ ,

$$\mathcal{M}^X(W_L(p_1)W_L(p_2)\dots) = \mathcal{M}_R^X(w(p_1)w(p_2)\dots)_R + \mathcal{O}\left(\frac{m_W}{E_i}\right). \quad (1)$$

In the traditional approach the subprocess cross section  $\sigma(W_LW_L \rightarrow W_LW_L)$  is convoluted with the effective  $W_LW_L$  luminosity [3] (which is a function of  $z = s_{WW}/s_{qq}$ ) to obtain the cross section for the  $WW$  fusion subprocess,

$$\sigma(qq \rightarrow qqW_LW_L) = \int dz \frac{d\mathcal{L}}{dz} \sigma(W_LW_L \rightarrow W_LW_L). \quad (2)$$

The traditional method is simple and effective but it neglects the transverse momentum of the final state  $q$  jets and the  $WW$  diboson. Knowledge of these transverse momentum distributions is important experimentally, e.g., to determine the efficiency of jet tag and veto detection strategies. In Higgs boson models the  $p_T$  distributions are readily obtained by computing the complete set of tree diagrams for  $qq \rightarrow qqWW$ , thus avoiding the EWA. It would be useful and

interesting to compute strong  $WW$  scattering cross sections in the same way. A U-gauge “transcription” to accomplish this was presented previously and verified by explicit computation for specific examples[4]. Here an algorithm is presented for  $s$ -wave scattering models to construct the complete family of gauge and Goldstone boson amplitudes in  $R_\xi$  gauge ( $WWWW$ ,  $WWWw$ ,  $WWww$ ,  $Wwww$ ) which follow from the initial Goldstone boson model amplitude  $\mathcal{M}_R^X(ww \rightarrow ww)$  by BRS invariance. The construction allows the gauge boson amplitude to be evaluated in any  $R_\xi$  gauge and in particular validates the U-gauge transcription.

In addition to providing information about the final state that is lost in the EWA, the new method is also more accurate since it correctly sums the Higgs sector signal and gauge sector background amplitudes coherently, while the EWA neglects the interference terms.<sup>3</sup> The transcription has other interesting consequences that will be discussed elsewhere: it reveals the “K-matrix” model, an *ad hoc* construction borrowed from nuclear physics to implement partial wave unitarity and chiral symmetry, as a (very!) nonstandard Higgs boson model, and allows a direct estimate of the effect of strong  $WW$  scattering on low energy radiative corrections.

#### The U-gauge transcription

The equivalence theorem, equation (1), already relates the gauge and Goldstone boson amplitudes, but not in a way that can be used to extract the  $WW$  fusion amplitude  $\mathcal{M}(qq \rightarrow qqWW)$  for strong  $WW$  scattering. The first step is to observe that to leading order in the  $SU(2)_L$  coupling  $g$  the on-shell  $WW$  amplitude is the sum of gauge sector and Higgs sector terms, e.g., in U-gauge

$$\mathcal{M}_{\text{Total}}^X(WW \rightarrow WW) = \mathcal{M}_{U,\text{Gauge}}(WW \rightarrow WW) + \mathcal{M}_{U,H}^X(WW \rightarrow WW). \quad (3)$$

and that at high energy,  $E \gg m_W$ , the gauge sector amplitude for longitudinal modes is dominated by its “bad high energy behavior”, a term growing like  $E^2$  which is at the same time the low energy theorem amplitude  $\mathcal{M}_{\text{LET}}$  of a strongly coupled Higgs sector[5, 1]. Using the equivalence theorem the U-gauge Higgs sector amplitude for model  $X$  is then

$$\mathcal{M}_{U,H}^X(W_L W_L \rightarrow W_L W_L) = \mathcal{M}_R^X(ww \rightarrow ww) - \mathcal{M}_{\text{LET}} + \mathcal{O}(g^2, \frac{m_W}{E}), \quad (4)$$

---

<sup>3</sup>The interference term is important when the gauge sector background is large, for instance, near Coulomb singularities.

which is just the Goldstone boson model amplitude  $\mathcal{M}_R^X(ww)$  with its leading threshold behavior subtracted.

This is still not in a form that can be readily embedded in  $\mathcal{M}(qq \rightarrow qqWW)$ . To proceed we limit the discussion to  $s$ -wave amplitudes and define an effective “Higgs” propagator  $P_X(s)$  by using the standard model Higgs sector amplitude for  $W^+W^- \rightarrow ZZ$  as a “template” for the effective theory,

$$\mathcal{M}_{U,H}^X(W_L W_L \rightarrow W_L W_L) = -g^2 m_W^2 \epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 P_X \quad (5)$$

where 1,2 denote the initial state bosons and 3,4 the final state. Here  $W_L W_L \rightarrow W_L W_L$  represents generically the two channels with  $s$ -wave threshold behavior,<sup>4</sup>  $W_L^+ W_L^- \rightarrow Z_L Z_L$  and  $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$ . For  $W^+W^+$  this is a big departure from the standard model since the  $s$ -channel exchange carries electric charge  $Q = +2$ , an effective “ $H^{++}$ ” exchange.

For simplicity I assume the weak gauge group is just  $SU(2)_L$  so that  $m_W = m_Z = gv/2$ . (I have verified that the conclusions do not depend on this assumption.) Then for  $E \gg m_W$  and up to corrections of order  $g^2$  the effective propagator is

$$P_X(s) = -\frac{v^2}{s^2} (\mathcal{M}_R^X(ww \rightarrow ww) - \mathcal{M}_{\text{LET}}). \quad (6)$$

The low energy theorem[5, 1] amplitudes are

$$\mathcal{M}_{\text{LET}} = \eta \frac{s}{v^2} \quad (7)$$

where  $\eta = +1$  for  $W^+W^- \rightarrow ZZ$  and  $\eta = -1$  for  $W^+W^+ \rightarrow W^+W^+$ . Notice that  $\mathcal{M}_{\text{LET}}$  contributes  $\pm 1/s$  to  $P_X$ , corresponding to a massless scalar pole, making explicit the connection between the spontaneously broken symmetry that implies  $\mathcal{M}_{\text{LET}}$  and the cancellation of the bad high energy behavior by Higgs boson exchange. The residual contribution to  $P_X$  from  $\mathcal{M}_R^X$  carries the model dependent strong interaction dynamics.

With the effective propagator  $P_X$  we can formulate the U-gauge transcription in a way that allows us to embed  $\mathcal{M}_{U,H}^X(W_L W_L)$  into  $\mathcal{M}(qq \rightarrow qqWW)$ . The prescription is simple: compute the usual gauge sector tree diagrams for

---

<sup>4</sup>Within these channels the restriction to  $s$ -wave models is not very onerous, since in these channels the LHC and electron colliders with energy  $\lesssim 3$  TeV will only probe energies for which the  $ff \rightarrow ffWW$  signals are dominantly in the  $WW$   $s$ -wave.

$\mathcal{M}(qq \rightarrow qqWW)$  in U-gauge but replace the Higgs boson exchange diagram(s) by  $s$ -channel exchange of  $P_X$  with the  $WW$  “ $H$ ” vertex given by  $gm_W g^{\mu\nu}$  as in equation (5). Here we make the usual, unavoidable extrapolation, required in any approach to strong  $WW$  scattering, from massless (in Landau gauge) Goldstone boson to  $m_W$  in the on-shell gauge boson amplitude to space-like  $-q^2 \simeq O(m_W^2)$  for the initial state virtual  $W$ ’s in the  $WW$  fusion amplitude. As always, the extrapolation contributes to the inevitable  $O(m_W/E)$  correction.

#### BRS and gauge invariance

In [4] the U-gauge transcription was verified by explicit calculation for the K-matrix model and the heavy Higgs boson standard model in the  $W^+W^+$  channel. We now demonstrate its validity in general for  $s$ -wave scattering by constructing a BRS invariant[6] set of amplitudes that relate the input model  $\mathcal{M}_R^X(w\bar{w} \rightarrow w\bar{w})$  to the corresponding gauge boson amplitude  $\mathcal{M}^X(W_L W_L \rightarrow W_L W_L)$ . Beginning from the Goldstone boson amplitude  $\mathcal{M}_R^X(w\bar{w} \rightarrow w\bar{w}) = \langle w\bar{w}w\bar{w} \rangle$  we use BRS invariance to construct the family of amplitudes  $\langle w\bar{w}w\bar{W} \rangle$ ,  $\langle w\bar{w}W\bar{W} \rangle$ ,  $\langle w\bar{W}W\bar{W} \rangle$  and  $\langle W\bar{W}W\bar{W} \rangle$  in the generalized  $R_\xi$  gauge. The gauge boson amplitude  $\mathcal{M}^X(WW \rightarrow WW)$  is then explicitly gauge invariant ( $\xi$  independent) and for the longitudinal modes is precisely the previously formulated U-gauge transcription. BRS invariance is verified explicitly for the *amplitudes*, even though there may ( $W^+W^- \rightarrow ZZ$ ) or may not ( $W^+W^+ \rightarrow W^+W^+$ ) be an underlying effective Lagrangian.

The construction of the BRS invariant set of amplitudes is accomplished by following a Feynman diagram algorithm, using as a template the set of diagrams for  $W^+W^- \rightarrow ZZ$  in the standard model. That is, we write each amplitude ( $\langle w\bar{w}w\bar{w} \rangle$ ,  $\langle w\bar{w}w\bar{W} \rangle$ ,  $\langle w\bar{w}W\bar{W} \rangle$ ,  $\langle w\bar{W}W\bar{W} \rangle$  and  $\langle W\bar{W}W\bar{W} \rangle$ ) as a sum of terms corresponding to the tree Feynman diagrams for the  $W^+W^- \rightarrow ZZ$  channel in the standard model. By maintaining the diagrammatic form of the amplitude and the essential relationships between vertices and propagators as they are in the standard model, we automatically preserve BRS invariance. This  $W^+W^- \rightarrow ZZ$  standard model template is applied to strong scattering in the  $W^+W^- \rightarrow ZZ$  channel and, less obviously, also to the  $W^+W^+ \rightarrow W^+W^+$  channel.

The diagrammatic algorithm is specified by the Feynman rules for vertices

and propagators. The  $WWH$  vertex<sup>5</sup> and the propagator  $P_X(s)$  were defined already in the U-gauge transcription, equations 5-6. The three and four gauge boson vertices and the propagators of the gauge and unphysical Goldstone bosons are determined by gauge sector dynamics and keep their standard model values.

The quartic Higgs sector coupling  $\lambda_X$  and the Goldstone-Higgs  $wwH$  vertex are related as in the standard model,  $\lambda_{wwH} = -2\lambda_X v$ . The quartic coupling  $\lambda_X$  is then determined by expressing the input model  $\mathcal{M}_R^X(ww \rightarrow ww)$  as the sum of the four-point contact interaction and  $s$ -channel Higgs exchange amplitude,

$$\mathcal{M}_R^X(ww \rightarrow ww) = -2\lambda_X \eta - (2\lambda_X v)^2 P_X \quad (8)$$

where  $\eta$  is defined in equation 7. Using equations (6) and (7) we solve equation (8) to obtain

$$\lambda_X = \frac{s}{2v^2} \frac{\mathcal{M}_R^X}{\mathcal{M}_R^X - \mathcal{M}_{\text{LET}}} \quad (9)$$

and

$$P_X = \frac{\eta}{s - 2\lambda_X v^2}. \quad (10)$$

While the effective coupling “constant”  $\lambda_X$  is not in fact constant but is in general a function of  $s$ , equation 10 shows that we have preserved (up to the factor  $\eta$ ) the standard model relationship between the effective Higgs propagator and the Higgs sector vertices which is crucial for maintaining BRS invariance.

The remaining interaction vertices are fixed by requiring that the  $WWWW$ ,  $WWWw$ ,  $WWww$ ,  $Wwww$ , and  $wwww$  amplitudes satisfy the BRS identities

$$(\partial W + \xi m_W w)^n = 0 \quad (11)$$

for  $n = 1, 2, 3, 4$ . For the  $W^+W^- \rightarrow ZZ$  channel all vertices not specified above are given precisely by their standard model values. The amplitudes obtained from our algorithm then trivially satisfy BRS invariance and  $\mathcal{M}(W^+W^- \rightarrow ZZ)$  is trivially gauge invariant ( $\xi$  independent in  $R_\xi$  gauge), because equation 10 assures that the necessary cancellations occur just as in the standard model.

It is less trivial but no less straightforward to verify that the prescription can be made to work for  $W^+W^+ \rightarrow W^+W^+$ . In this case it is necessary to define

---

<sup>5</sup>  $WWH$  denotes generically  $W^+W^-H$  and  $ZZH$  with reference to the  $W^+W^- \rightarrow ZZ$  channel and  $W^+W^+H^{++}$  for  $W^+W^+ \rightarrow W^+W^+$ .

some nonstandard interaction vertices, since the U-gauge transcription defined above already mutilates the standard model structure for  $W^+W^+ \rightarrow W^+W^+$  by substituting an  $s$ -channel effective  $H^{++}$  exchange for the  $t$  and  $u$ -channel  $H^0$  exchanges of the standard model. Clearly all interactions of the effective  $H^{++}$  boson are nonstandard and require definition. The U-gauge transcription already specifies the  $H^{++}W^-W^-$  vertex as  $gm_W g^{\mu\nu}$  (see equation 5), equal<sup>6</sup> to the standard model  $HW^+W^-$  vertex. The remaining nonstandard vertices are fixed by insisting on the validity of the BRS identities, equation 11, applied to  $\langle W^+W^+W^-W^- \rangle$  for  $n = 1,2,3,4$ . With the vertices chosen to satisfy BRS invariance we find that  $\xi$  independence of  $\mathcal{M}(W^+W^+ \rightarrow W^+W^+)$  in  $R_\xi$  gauge is also automatically assured.

In addition to defining the interactions of the effective  $H^{++}$  boson we must adopt nonstandard quartic couplings for the  $w^+w^+w^-w^-$  and  $W^+W^+w^-w^-$  vertices: the former is  $-1/2$  times its standard model value while the latter does not exist at all in the standard model. Vertices that do not exist in the standard model or that differ from their standard model values are given in table 1.

To illustrate how the diagrammatic algorithm satisfies BRS invariance, consider the identity equation 11 with  $n = 2$ , applied to the two initial state bosons in  $WW$  scattering. We can consider  $W^+W^- \rightarrow ZZ$  and  $W^+W^+ \rightarrow W^+W^+$  concurrently, since in our approach they are given by the same set of Feynman diagrams. The BRS identity for the scattering amplitude  $W_1W_2 \rightarrow W_3W_4$  is

$$\epsilon_{3\alpha}\epsilon_{4\beta} \left( k_{1\mu}k_{2\nu}\mathcal{M}^{\mu\nu\alpha\beta} + im_W(k_{1\mu}\mathcal{M}_{w_2}^{\mu\alpha\beta}k_{2\nu}\mathcal{M}_{w_1}^{\nu\alpha\beta}) - m_W^2\mathcal{M}_{w_1w_2}^{\alpha\beta} \right) = 0. \quad (12)$$

The subscript  $w_i$  indicates the amplitude in which gauge boson  $W_i$  is replaced by Goldstone boson  $w_i$ .

Using the Feynman rules defined above to evaluate the amplitudes in equation 12 in  $R_\xi$  gauge, we find after trivial cancellations that the remaining terms are

$$\delta_{\text{BRS}}^2 = \frac{1}{2}g^2m_W^2\epsilon_3 \cdot \epsilon_4 \left( (s - 2\lambda_X v^2)P_X - \eta \right). \quad (13)$$

Using equation 10 the right side vanishes, confirming the BRS identity equation 12. All other BRS identities can be similarly verified.

---

<sup>6</sup>I follow the phase conventions of the CORE compendium.[7]



### Discussion

The U-gauge transcription has been verified for  $s$ -wave strong  $WW$  scattering models by demonstrating its consistency with BRS invariance for the complete set of gauge and Goldstone boson scattering amplitudes. A Feynman diagram algorithm was defined, including an effective “Higgs” propagator that carries the strong scattering dynamics and a related energy dependent “effective”  $\phi^4$  Higgs sector coupling constant. The transcription is useful both in high energy applications, to strong  $WW$  scattering at  $pp$  and  $e^+e^-$  colliders, and in low energy applications, to estimate the effect of strong  $WW$  scattering on electroweak radiative corrections.

As discussed in [4] the U-gauge transcription is more accurate and more complete than the effective  $W$  approximation for computing strong  $WW$  scattering. It is more accurate because it retains the interference between the strong  $WW$  scattering amplitude and the gauge sector background amplitude, and also because it provides the transverse momentum of the  $WW$  diboson which is neglected in the EWA. It is more complete because it provides the full three-momentum distribution for the final state quark jets in the  $qq \rightarrow qqWW$  process, while the EWA neglects the jet transverse momenta (also introducing an error in the determination of the jet rapidities).

The final state jet distributions are needed to compute the efficiency of detection strategies such as the central jet veto[8] and the forward jet tag[9]. The former is very effective against the gluon exchange and electroweak gauge sector backgrounds, and the latter may be very useful against the surprisingly large  $\bar{q}q \rightarrow WZ$  background[10] to the  $W^+W^+ \rightarrow W^+W^+$  strong scattering signal. In previous studies the necessary jet distributions have been estimated assuming the same shape for strong scattering as for the standard model with a heavy (typically 1 TeV) Higgs boson. This assumption can now be tested using the U-gauge transcription. I find that it works well at low enough energy colliders, for which the strong scattering and heavy Higgs cross sections are “squashed” into roughly the same region in  $s_{WW}$ , but not at higher energy colliders with enough phase space to allow the differences in the  $WW$  energy spectrums to emerge. From this perspective the LHC is a “low” energy collider, while at SSC energies (R.I.P.) the differences begin to be important.

The effective Higgs sector propagator defined in the U-gauge transcription may also be used to estimate the direct effect of strong  $WW$  scattering on low energy radiative corrections. Unlike the typically large corrections predicted by technicolor models, the correction due just to strong nonresonant dynamics in  $WW$  scattering is not very much bigger than the effect of the 1 TeV standard model Higgs boson. These results and other applications of the U-gauge transcription will be presented elsewhere.

Acknowledgements: I wish to thank R.N. Cahn M.K. Gaillard, and H. Murayama for helpful discussions and J. Ellis for perusing the manuscript. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contracts DE-AC03-76SF00098 and DE-AC02-76CHO3000.

## References

- [1] M.S. Chanowitz and M.K. Gaillard, *Nucl. Phys.* B261, 379 (1985).
- [2] J.M. Cornwall, D.N. Levin, and G. Tiktopoulos, *Phys. Rev.* D10, 1145 (1974); C.E. Vayonakis, *Lett. Nuovo Cim.* 17, 383 (1976); B.W. Lee, C. Quigg, and H. Thacker, *Phys. Rev.* D16, 1519 (1977); M.S. Chanowitz and M.K. Gaillard, *op cit.*; G. Gounaris, R. Kögerler, and H. Neufeld, *Phys. Rev.* D34, 3257(1986); H. Veltman, *Phys. Rev.* D41, 2294(1990); J. Bagger and C. Schmidt, *Phys. Rev.* D41, 264 (1990); W. Kilgore, *Phys. Lett.* 294B, 257(1992); H-J. He, Y-P. Kuang, X-Y. Li, *Phys. Rev. Lett.* 69, 2619 (1992) and *Phys. Rev.* D49, 4842 (1994).
- [3] M.S. Chanowitz and M.K. Gaillard, *Phys. Lett.* 142B, 85 (1984); G. Kane, W. Repko, B. Rolnick, *Phys. Lett.* B148, 367 (1984); S. Dawson, *Nucl. Phys.* B29, 42 (1985).
- [4] M.S. Chanowitz, hep-ph9512358 *Phys. Lett.* B373, 141 (1996).
- [5] M.S. Chanowitz, H.M. Georgi, M. Golden, *Phys. Rev.* D36, 1490 (1987); *Phys. Rev. Lett.* 57, 2344 (1986).

- [6] C. Becchi, A. Rouet, R. Stora, *Phys. Lett.* 52B, 344 (1974); *Commun. Math. Phys.* 42, 127 (1975).
- [7] V. Borodulin, R. Rogalyov, S. Slabospitsky, IHEP-95-90, hep-ph/9507456.
- [8] V. Barger et al., *Phys. Rev.* D42, 3052 (1990).
- [9] R.N. Cahn, S.D. Ellis, and W.J. Stirling, *Phys. Rev.* D35, 1626 (1987).
- [10] G. Azuelos, C. Leroy, and R. Tafiout, ATLAS Internal Note PHYS-NO-033, 1993 (unpublished); G. Azuelos, talk presented at CERN, April 21, 1994 (unpublished); M. Chanowitz and W. Kilgore, hep-ph/9412275, *Phys. Lett.* B347, 387 (1995); J. Bagger et al., hep-ph/9504426, *Phys. Rev.* D52, 3878 (1995).

Table 1. Nonstandard interaction vertices for  $W^+W^+ \rightarrow W^+W^+$  scattering. Also shown are analogous vertices for  $W^+W^- \rightarrow ZZ$ , which agree precisely with the standard model. All momenta are inflowing, phase conventions are as in the CORE compendium[7], and  $\eta$  is defined below equation 7.

$W^+W^+ \rightarrow W^+W^+$	$W^+W^- \rightarrow ZZ$	Interaction
$H^{++}(k)W_\mu^-(p)W_\nu^-(q)$	$H^0(k)W_\mu^+(p)W_\nu^-(q)$	$gm_W g^{\mu\nu}$
$H^{++}(k)W_\mu^-(p)w^-(q)$	$H^0(k)W_\mu^+(p)w^-(q)$	$ig(q^\mu - k^\mu)/2$
$H^{++}(k)w^-(p)w^-(q)$	$H^0(k)w^+(p)w^-(q)$	$-2\lambda_X v$
$w^+(p_1)w^+(p_2)W_\mu^-(p_3)W_\nu^-(p_4)$	$w^+(p_1)w^-(p_2)Z_\mu(p_3)Z_\nu(p_4)$	$\eta g^2 g^{\mu\nu}/2$
$w^+(p_1)w^+(p_2)w^-(p_3)w^-(p_4)$	$w^+(p_1)w^-(p_2)z(p_3)z(p_4)$	$-2\eta\lambda_X$